# MULTI-OBJECTIVE OPTIMISATION OF PID AND H$\infty$ FIN/RUDDER ROLL CONTROLLERS

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Abstract: Active control of ship roll is necessary for operability of an important number of ships. As such it has been strongly developed in the past twenty years. A way of improving performances is to use and control rudders as well as fins. A MIMO control law synthesis methodology is presented in this paper, which is based on multi-objective optimisation. The optimisation is realised with a genetic algorithm. It is applied to a PID and a $H_\infty$ controller, both MIMO. Simulation results with various speeds are given. 

Keywords: Ship control; roll stabilisation; multi-objective optimisation; PID control; H infinity control;

## 1. INTRODUCTION

Sea-keeping abilities determines the operability of numerous ships such as military vessels (operability, aircraft landing, crew comfort) and passenger vessels for passengers' comfort and security. Dampening movements, especially roll, pitch and heave, by passive and much more by active stabilisation systems drastically improves the operability.

Many ships have such a system installed onboard, usually quite efficient and simple. Different ways of improving the performances of the stabilisation systems are being investigated. Among them is the ability of the control system to adapt to the environmental conditions, defined by parameters like ship speed, wave encounter angle, wave height, wave frequency, loading conditions. Before achieving this, it is required to have a method for designing efficient roll controllers in a MIMO context at each operating point, considering these parameters are fixed. This first step of the work towards gain-scheduled control is here dealt with considering ship speed.

This paper is organised as follows. The process is described in section 2 as a MIMO system. The methodology is detailed in section 3, which uses multi-objective optimisation to tune a generic controller. It is preceded by section 3.1 which lists the main visible results in the literature. The results of the synthesis for MIMO PID and $H_\infty$ controllers for different ship speeds are given in section 4. Section 5 gives the perspectives.

## 2. MODEL

The dynamics of a ship in a seaway, under the linearity assumption, is generally divided into two different parts: first how the ship reacts to its actuators (fins and rudders); second, to the environment, that is to say the wind, waves and current - only waves are considered here. The effects of both are then added, be it expressed in efforts or in movements. It is assumed they can be computed separately from one another [1]. The model is then considered to be the superposition of the motions due to the waves and the motions due to the actuators. So, waves will be taken as a

[1] This is not valid from a hydrodynamics point of view. Yet it is a useful and quite correct assumption when applying linear control theory.

disturbance additive on the outputs of the ship's dynamics, as shown on figure 1.

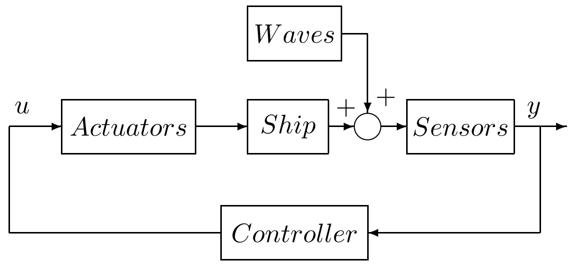


Fig. 1. Control model with output additive disturbance

In addition, it is assumed that the ship has no list, and that no non linear phenomenon appears. The ship's model is supposed to depend only on speed relative to water.

### 2.1 *Ship dynamics*

The ship is fitted with rudders which are used for stabilisation and stabilisation fins; they have different effects on the ship's motions. The fins are used only for roll stabilisation, and should interfere very little with the heading. On the contrary, rudders have a great influence on roll motions, but are primary used to control the yaw. Yet these effects appear in two quite separated frequency ranges.

The inputs $u$ for the model are the realised actuators position: $\alpha$ for the fins, and $\delta$ for the rudders. $x$ is the state of the ship, composed of sway velocity $v$, roll velocity $p$, roll angle $\phi$, yaw velocity $r$ and yaw $\psi$ ($x = [v\, p\, \phi\, r\, \psi]^T$). The dynamics of the ship on calm sea is modeled by equation 1 and 2. The wave disturbance $w$ only affects the dynamical system output in $\phi$, $p$, $\dot{p}$ and $\psi$. $V$ is the value of the speed of the ship relative to water. The matrices $B$ and $C$ depend on the speed as the roll acceleration is measured.

$$\dot{x} = A(V)x + B(V)u \tag{1}$$

$$y = C(V)x + D(V)u + w \tag{2}$$

The model is parameterised in speed $V$. The coefficients of the matrices are dependent on $V$ as second (fins and rudders efficiency, damping), first or zeroth (buoyancy) order polynomials.

The measures $y$ considered for control (see equation 3) are the roll angle $\phi$, the roll velocity $p$, the roll acceleration $\dot{p}$ and the heading angle $\psi$.

$$y = \begin{pmatrix} \phi + \phi_w \\ p + p_w \\ \dot{p} + \dot{p}_w \\ \psi + \psi_w \end{pmatrix} \tag{3}$$

### 2.2 *Actuators*

The actuators have a dynamics, that can be modeled by a second order transfer function as in equation 4.

$$\alpha(s) = \frac{\omega_a^2}{\omega_a^2 + 2\zeta_a\omega_a s + s^2}\alpha_d(s) \tag{4}$$

where $\alpha_d$ is the desired, and $\alpha$ is the realised actuator position.

The parameters $\omega_a$ and $\zeta_a$ may be determined by experimental measures. Position and speed of both the rudders and fins are not free. The device itself limits the available position of the fins and rudders. But in addition, as cavitation and mechanical efforts increase with speed, they may become intolerable; the saturation of the fins in position is dependent on speed. Its dependency is usually realised in $1/V^2$, see Lloyd (1989). The other saturation do not necessitate variation of their values.

### 2.3 *disturbance*

The disturbance is calculated from the seakeeping characteristics of the ship and the sea state spectrum. The sea state spectrum used here is the Bretschneider spectrum (Fossen, 1994):

$$S_w w(\omega) = 0.31 H_s^2 \frac{\omega_0^4}{\omega^5} \exp\left(-1.25\frac{\omega_0^4}{\omega^4}\right) \tag{5}$$

$H_s$ is the significant wave height (mean of the height of highest third of waves), and $\omega_0 = \frac{2\pi}{T_0}$ the dominating wave pulsation.

### 2.4 *Simulation model*

The previous sections detail the model used for synthesis. A more realistic model was used to test the control laws. The simulation model still comprises an output additive disturbance, generated in position (and attitude), speed and acceleration. In addition, it takes into account the temporal non-linear aspects of saturation (in angle and rate for both the fins and rudders) and a comprehensive actuators dynamics, and digitalisation of the control law. In addition to this, a pure delay is added in temporal simulations to make up for the information transportation effects in the ship internal network.

## 3. SYNTHESIS METHODOLOGY

### 3.1 *Review of common synthesis methods*

Different control synthesis techniques have been used so far. First methods were based on PID control with feedback in roll, roll speed and roll acceleration. Other forms of control law also tested are: LQG (Sgobbo and Pearsons, 1999; van Amerongen *et al.*, 1990), neural networks (Liut *et al.*, 2000), pole placement for RRS (Fossen, 1994), sliding mode (Lauvdal and Fossen, 1996). Hearns et al. (2000) proposed a robust controller design through QFT.

The choice of PID coefficients is based on the phase compensation of delays induced by actuators servos, and on statistical measures of actuators use on a standard sea state (Lloyd, 1989). This give good results as accounted for in the literature. Katebi et al. (2000) exposed a PID tuning framework using optimisation. They also proposed $H_\infty$ control laws which weights are based on previous works of Grimble et al (1993).

In the spirit of Katebi et al. (2000), PID and $H_\infty$ controller are studied. More precisely, here, a common scheme of optimisation is used to tune controllers of different natures. In the first case this will directly give the PID coefficient. In the second case this will give the $H_\infty$ weights that are parameterised by a finite (and small) number of coefficients; the final calculation of the $H_\infty$ controller being classical.

The proposed methodology is described in the following paragraphs.

### 3.2 *Principles*

The stabilisation problem expressed in common language terms is: 'use at best the actuators to stabilise the roll, but do not destabilise the yaw nor use too much energy'. In addition to these three requirements, are requirements common in the control field (stability, robustness). This is expressed in the following paragraph.

### 3.3 *Specifications*

There are primary two antagonistic objectives:

**O1** Reduce the roll motion
**O2** Use the minimal quantity of energy

The description of the computation is given in the next paragraph. The second objective is necessary to ensure that the two actuators do not compensate for one another, case which may appear.

This defines a two-goals optimisation problem ; it is tractable conveniently with stochastic algorithms such as genetic algorithms. It could have been possible to aggregate the different objectives into a unique one typically by linear combination, but consequently loosing the grasp on the optimisation meaning. It has then been decided to keep the different objectives, so as to make the most enlightened choice possible.

The wish not to destabilise the yaw motion while damping roll motions can be understood by a relative use of rudders less than of the fins. This implies a constraint on the relative efficiency of both actuators on the roll reduction. But it has been difficult to obtain solutions with this configuration of objectives, and another objective was added to express this constraint.

**O3** Respect as precisely as possible the repartition constraint

In addition to this, there are constraints that have to be respected, and which correspond to usual stability and robustness constraints, but also to specific requests of the problem:

**C1** Controller stability
**C2** Closed loop stability under a given control application delay
**C3** Acceptable delay margin
**C4** Low amplification under and over resonance (two constraints)
**C5** Limited saturations for fins and rudders in both position and velocity (four constraints)

### 3.4 *Calculations*

3.4.1. *Objectives* The first objective is expressed as the minimisation of the roll RMS value on a particular sea state for the closed loop system. The second objective is the sum of the RMS values of the fins' and rudders' positions (resp. $\sigma_\alpha$ and $\sigma_\delta$), for the same sea-state.

Objective $O3$ is evaluated from the RMS values of the actuators positions. The repartition, noted $r$, is defined (see equation 6) as the weighted ratio 'use of the fins' over 'total use of the actuators', the weights being the $H_\infty$ norm of the open loop transfer functions between fin ($N_\alpha$) and rudder ($N_\delta$) position and roll.

$$r = \frac{N_\alpha \sigma_\alpha}{N_\alpha \sigma_\alpha + N_\delta \sigma_\delta} \tag{6}$$

3.4.2. *Stability constraints* The stability of the controller is tested because of numerical uncertainties during the synthesis that may lead it to be unstable.

The closed loop stability is tested through the calculation of the closed loop poles of the system, given a delay in the application of the computed control. The delay is approximated by a Padé, and simulates the presence of unmodeled phase, digitalisation and information transfer. The delay margin itself being more precisely evaluated with $\mu-$analysis, as the control problem is MIMO.

3.4.3. *Low amplification constraint* Constraints C4 are calculated from sensibility transfer between perturbation and output (in roll rotation speed). They require the closed loop not to amplify roll too much outside the resonance zone (where it dampens the roll).

3.4.4. *Actuators saturation constraints - C5* The saturations are impossible to evaluate precisely when working in the frequency domain. It has only a temporal meaning but may be transposed to the frequency domain under statistical assumptions.

Under reasonable assumptions [2] the expression of the expected frequency of threshold crossing (at value $\alpha_0$) in a period of one minute, is:

$$e^+ = \frac{60}{2\pi}\sqrt{\frac{m_2}{m_0}}\exp(-\frac{2\alpha_0^2}{m_0}) \tag{7}$$

### 3.5 *Implementation*

Multi objective optimisation is conveniently tractable through a genetic algorithm, though it sometimes is slow. They are based on the evolution of 'individuals' representing potential solutions; at each iteration, the algorithm processes the genes through different operations (selection, mutation, cross-over) to produce a new generation of individuals. The algorithm stops when a given number of generations has been achieved.

The outcome of a multi objective problem is not a unique solution, but a set of solution, each being adequate as no supremacy of an objective above another has been set. This set is called the Pareto front. The tricky point is then to choose a particular solution fitted to our problem. The choice is made by the following algorithm. Choose the solution:

(1) that belongs to the Pareto front or is very near
(2) that complies to the repartition constraint
(3) that has the best roll reduction

The software used for this work is modeFRONTIER [3]. It is a conception optimisation software, with a dedicated user interface, several optimisation methods supporting mono or multi-objective designs with constraints.

[2] The signals are centered, and have narrow band spectra, and the amplitude repartition of the signals with time respects a Reynolds law (Price and Bishop, 1974; Lloyd, 1989)

[3] Developped by ESTECO, see www.esteco.com

## 4. CONTROL LAWS CALCULATIONS AND SIMULATION RESULTS

The available measures are the roll angle, the roll velocity, the roll acceleration and the heading angle.

### 4.1 *PID*

The expression of the control law is :

$$\begin{aligned}\alpha &= \left(K_{1\alpha} + sK_{2\alpha} + s^2K_{3\alpha}\right)\phi \\ \delta &= \left(K_{1\delta} + sK_{2\delta} + s^2K_{3\delta}\right)\phi + \\ &\quad \left(s^{-1}K_{4\delta} + K_{5\delta} + sK_{6\delta}\right)\psi\end{aligned}$$

The coefficients to be optimised are $K_{i\alpha}$ and $K_{i\delta}$ for $i \in \{1..3\}$. The coefficients $K_{i\alpha}$ for $i \in \{4..6\}$ are fixed using a simple pole placement method, see (Fossen, 1994). The simulation results are given in tables 1, 2 and 3.

### 4.2 $H_\infty$

The $H_\infty$ controller synthesis is based on the minimisation of the infinity norm of a given transfer function between weighted input disturbances and weighted controlled outputs. The weights are dynamical (they have internal states) and are used to shape the closed loop transfer functions.

The choice of the weights and the description of the closed loop system is critical for the results. Furthermore, they have to be parameterised for optimisation.

The synthesis principle is given in figure 2. The following paragraph describes the weights.

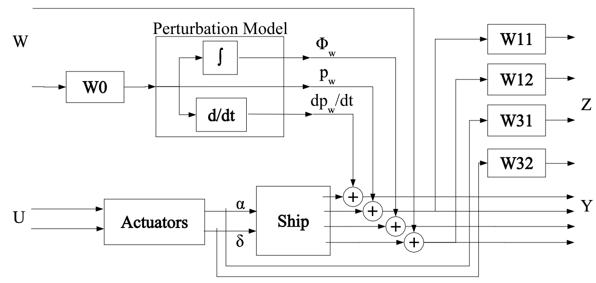


Fig. 2. $H_\infty$ standard synthesis model

The different signals disturbing the system are not independent of each other. Only two perturbation signals are used: $p_p$ and $\psi_p$, the rest of the signals being calculated from them by approximated integration and derivation, as shown in the perturbation model in figure 2.

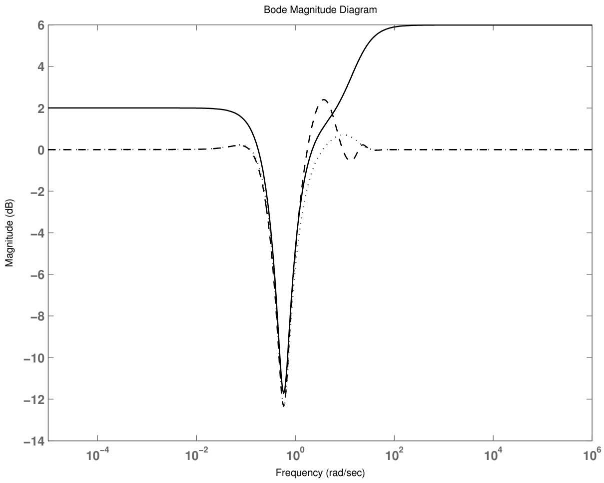


Fig. 3. $H_\infty$ roll sensibility weight of order 3 (solid), closed loop sensibility without input delay (dotted) and with input delay (dashed)

4.2.1. *Weights* The choice of the weights determine the shape of the sensibility transfer (between disturbance $w$ and the output $z$). The final aim is to damp the wave induced roll, to keep the actuators efficient and not to saturate them too much (see section 3.3).

The different weights are taken as follows:

**W0** Filter on the perturbation input, low pass
**W11** Roll sensibility weight - see section 4.2.2
**W12** Yaw sensibility weight;
$W_{12}(s) = K_r \frac{\tau_{r1}s+1}{\tau_{r2}s+1}$, with $\tau_{r2} > \tau_{r1}$
**W31** Fin disturbance sensibility weight;
$W_{31}(s) = K_f \frac{\tau_{f1}s+1}{\tau_{f2}s+1}$ with $\tau_{f2} > \tau_{f1}$
**W32** Rudder disturbance sensibility weight;
$W_{32}(s) = K_r$

The parameters used for optimisation are $K_r$, $K_f$ and the maximal attenuation of roll in the roll sensibility weight. Dynamics in the yaw sensibility weight is meant to compensate in yaw only the low frequency motions; for the fin disturbance sensibility weight, it is meant to avoid using the fins at low frequencies.

4.2.2. *Roll sensibility weight* This weight determines the obtained performance. As the best performance is to be obtained, it is parameterised so as to be optimised. Different shapes were tested, of different orders. A simple and efficient solution is determining the shape by interpolation from a given number of control points. The figure 3 gives an idea of the shape of this weight for order 3 (solid) and realised closed loops without (dotted) and with (dashed) a delay on the input.

The order of the weight influences the achievable performance as with more parameters it is more easily malleable. But as order of the standard model for $H_\infty$ increases the synthesis requires more time.

Table 1. Roll damping performance (%)

| Speed | $PID$ | $H_\infty 1$ | $H_\infty 2$ |
|---|---|---|---|
| 10 | 56 | 54 | 45 |
| 15 | 76 | 77 | 63 |
| 20 | 78 | 82 | 77 |
| 25 | 72 | 83 | 71 |

Table 2. Fin usage (degrees RMS)

| Speed | $PID$ | $H_\infty 1$ | $H_\infty 2$ |
|---|---|---|---|
| 10 | 5.81 | 6.45 | 8.57 |
| 15 | 4.55 | 4.97 | 5.35 |
| 20 | 3.71 | 4.54 | 4.95 |
| 25 | 2.08 | 3.11 | 3.49 |

4.3 *Simulation results*

The controllers calculated have been tested on a frigate-like ship (length 120m, displacement 3000 tons). The loading conditions were sea state 5 (Hs=3.25m & T0=9.7s) for an encounter angle of 90. The simulation duration is 20 minutes. The control laws were digitalised and applied at a sampling time of 0.1 second.

The obtained results are given in terms of quality, that is the roll reduction rate of the stabilised ship compared with the unstabilised ship, see table 1. Tables 2 and 3 give the RMS values of the actuators position in degrees.

There is a destabilising effect on yaw control. The RMS yaw signal is increased by 5 to 25% at all speeds for the $H_\infty$ controllers, whereas, for PID, it is increased at lower speeds (10 and 15 knots) and decreased (stabilised) at higher speeds (20 and 25 knots). This behaviour stems from the tuning differences between the controller: the yaw control bandwidth for the PID is higher than for the $H_\infty$ controllers.

The $H_\infty$ controller with an order 5 roll sensibility weight ($H_{\infty 1}$ in the tables) gives better results at higher speeds, while the $H_\infty$ controller with an order 3 roll sensibility weight ($H_{\infty 2}$ in the tables) gives equivalent results as the MIMO PID controller. Attention must be paid at the fact that the outcome of the optimisation may be improved, in the three cases: longer optimisation could lead to equivalent performance between PID and $H_\infty$. Furthermore, the order of the $H_\infty$ controllers is 18 or 20; the complexity induced has to be taken into account as it implies problems of digitalisation and calculations onboard; difficulties finding the appropriate digitalisation method were encountered when applying the controllers in simulation.

A typical simulation is given in figure 4 for speed 25 knots.

Table 3. Rudder usage (degrees RMS)

| Speed | $PID$ | $H_\infty 1$ | $H_\infty 2$ |
|---|---|---|---|
| 10 | 2.14 | 2.55 | 2.42 |
| 15 | 1.91 | 2.10 | 1.73 |
| 20 | 0.72 | 1.41 | 1.70 |
| 25 | 0.69 | 0.92 | 1.07 |

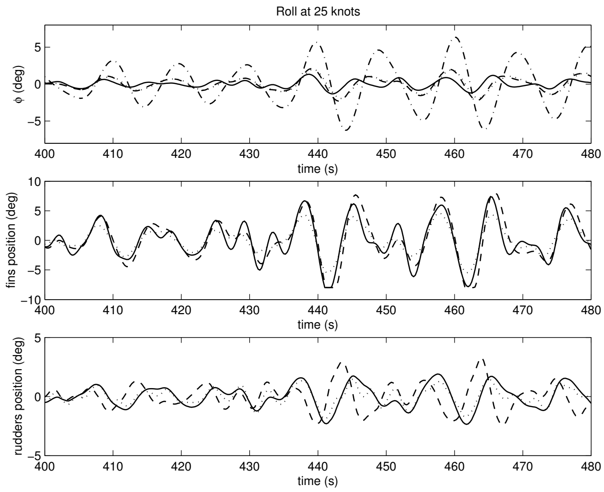


Fig. 4. Simulation for different controllers at speed 25 knots (dash-dotted: unstabilised ship; solid: $H_{\infty 1}$ control; dashed: $H_{\infty 2}$ control; dotted: PID control)

## 5. PERSPECTIVES

The control synthesis problem solved here is not an end in itself. The main purpose of this work is to synthesise gain-scheduled controllers. Usually done with PID, the coefficients are reduced with speed increase, see (Lloyd, 1989). The goal is to take advantage of the $H_\infty$ synthesis method for its results, and also of the rigourous construction of a gain-scheduled controller via LMIs. Indeed, no theoretical result can prove that the gain-scheduled PID controller stabilises the system. By contrast, gain-scheduled controllers synthesised via LMIs are proven to stabilise the system.

Contrary to the common method when controllers are interpolated, it is here the weights that have to be interpolated, and parameterised with the scheduling variables as a polytopic or an LFT model, see (Biannic, 1996; Duc and Hiret, 2001). The controller is then generated from the resolution of an optimisation problem under LMI constraints.

## 6. CONCLUSION

In this article is described the methodology used to tune MIMO PID and $H_\infty$ controllers for the roll damping of a ship. The method is multi-objective, and searches the best performance achievable, while respecting stability, robustness and physical constraints. The simulation results show the interest of $H_\infty$ controllers as giving better or equivalent roll reduction results if finely exploited, though inducing problems of complexity and digitalisation. Furthermore, the rigourous approaches to gain-scheduled controllers linked to $H_\infty$ give interesting perspectives.

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